Understanding the physics of meteoritic descent

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The fall of a body through a planetary atmosphere is analyzed in a relatively simple, essentially analytic, manner, emphasis being placed on the physical processes involved. © *1995 American Association of Physics Teachers.*

I. INTRODUCTION

The collision of a series of massive cometary objects with Jupiter has rekindled interest in the problem of atmospheric entry: what happens to an object impinging on a planetary atmosphere?

A thorough treatment of atmospheric entry looks like a prime candidate for numerical simulation; however, the basic qualitative conclusions can be arrived at analytically by means of a simple but very useful artifice, while approximate quantitative assessments require only the services of a reasonably complete hand calculator. This allows one to concentrate on a number of interesting physical phenomena, and indeed to suspect that since the values of certain critical parameters are taken virtually *ad hoc*, the construction of a detailed numerical model constitutes little more than an interesting computer physics exercise.

II. THE FLIGHT OF A METEORITE

Consider an object of mass m (we shall refer to it as a "meteorite" for linguistic convenience; it could just as well be an asteroid or a comet) passing through a planetary atmosphere towards the surface. At an altitude y, it will be subject to a number of forces: *planetary gravity*—I shall write the acceleration due to gravity as g; *atmospheric drag*—I shall write this as D; and the centrifugal force arising from motion around the planetary center.

We note first that a planet is a large body compared to the altitude over which its atmosphere is significant. This allows one at the outset to introduce two important simplifications. First, the curvature of the surface will be neglected, and the fall will be analyzed in a Cartesian coordinate system, the x-axis lying in the plane of the now flat planet, the altitude y thus becoming the vertical coordinate. Second, the gravitational acceleration can be taken to be a constant, equal to its superficial value.

Suppose that an altitude y the meteorite is moving at velocity v in a direction inclined at an angle ψ with respect to the vertical. The motion is determined by the following.

(1) The horizontal velocity component

$$\frac{dx}{dt}=-v\sin\psi. \tag{1}$$

(2) The vertical velocity component

$$\frac{dy}{dt}=-v\cos\psi. \tag{2}$$

(3) The balance of forces along the velocity vector

$$m\frac{dv}{dt}=-D+mg\cos\psi-\frac{m(dx/dt)^2}{R+y}\cos\psi, \tag{3}$$

where R is the planetary radius. The third term on the right-hand side takes into account the centrifugal force due to motion around the planetary center; we can in this way (if we wish) take proper account of all the forces, while continuing to use the flat planet approximation.

(4) The balance of forces perpendicular to the velocity vector

$$mv\frac{d\psi}{dt}=mg\sin\psi-\frac{m(dx/dt)^2}{R+y}\sin\psi. \tag{4}$$

The second term of this equation again takes account of the centrifugal force.

A number of further simplifications are in order.

We are assuming that the moving body does in fact descend; this implies that the centrifugal force must be smaller than the weight, and we shall in fact assume that it is small enough to be negligible. Consequently, Eqs. (3) and (4) become

$$m\frac{dv}{dt}=-D+mg\cos\psi, \tag{5}$$

$$mv\frac{d\psi}{dt}=mg\sin\psi. \tag{6}$$

Now, entry velocities are typically in the tens of km/s, while gravitational accelerations are of the order of 10 m/s^2; consequently, over most of the flight trajectory g/v is a negligible number of inverse seconds, and we can consider the flight to occur at essentially constant angle ψ since ballistic descent times are never very long. This of course will not be true close to the planetary surface; therefore the details of what happens at low altitudes and for small velocities should be taken with a pinch of salt.

The drag force D is produced by two distinct processes.

(1) The air flow in the immediate vicinity of a moving surface must be at rest with respect to it; viscous interactions act therefore as a force opposing motion, while transferring energy to the surface, heating it; this is known as "slip friction" in the aerodynamic community, and is obviously a rising function of the surface over which the air flow occurs.

(2) As air flows around the moving obstacle, a pressure difference builds up fore-aft, again opposing motion. This is often called "pressure drag."

The relative importance of these two components is a function of the shape of the obstacle; slip friction will be relatively small for a short disc moving perpendicular to its surface, but relatively large for a wide, flat winglike structure moving in a direction parallel to the surface.

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One expects that for an irregular object such as a meteorite, the pressure drag will be the dominant factor. Consider a surface oriented perpendicular to the velocity vector; in the most extreme case, atmospheric molecules will be reflected from the surface, perfectly and perfectly perpendicular, so that (neglecting what happens behind the disc) the drag is just the rate of transfer of momentum from the atmosphere to the object

$$D=2\rho_a v^2 S, \tag{7}$$

where ρ_a is the atmospheric density and S the cross-sectional surface area.

In practice, of course, material flows around the body, and pressure builds up behind, reducing the fore-aft pressure difference; the pressure drag is more generally written as

$$D=\tau\rho_a v^2 S, \tag{8}$$

where $\tau \leqslant 2$ takes into account the complexities of turbulent flow around an arbitrary obstacle, and is a function of velocity. At the speeds in which we shall be interested (largely supersonic):

$$\tau \rightarrow 0.5.$$

Readers might note that in the meteoritic and aerodynamic community, the right-hand side of Eq. (8) is conventionally divided by two, so that the conventional drag parameter is twice as large as the value of τ I shall be using here. To minimize the bookkeeping of unimportant constants, I shall not follow this convention here. And in the same spirit, I shall consider in the rest of this paper a cubic body, so that

$$S=(m/\rho)^{2/3}, \tag{9}$$

where ρ is its density; an unimportant factor close to 1 multiplies the right-hand side of Eq. (9) for noncubical bodies whose principal axes are all of the same order of magnitude.

Now, within an atmosphere and for masses less than, say, a few million kg, so long as the velocities are in the km/s range or over we can see that the drag force is much more important than the weight, allowing one to simplify the equations still further by neglecting the weight term in Eq. (5). Even for much larger masses, this term is of little importance, since its cumulative effect is small compared to the cumulative effect of the drag over the distance that a projectile travels in the atmosphere.

The drag force is a function of the atmospheric density ρ_a; in an isothermal atmosphere, the density follows an exponential law

$$\rho_a=\rho_0 \exp(-y/H), \tag{10}$$

where H is called the scale height and is equal to $kTR^2/GM\mu$; k and G are, respectively, Boltzmann's constant and the gravitational constant, T is the temperature, M is the mass of the planet, and μ is the molecular weight of the atmosphere. ρ_0 is the atmospheric density at the planetary surface. Planetary atmospheres are not isothermal; nevertheless, it is useful to characterize an atmosphere over a certain altitude range by an exponential law of the type of Eq. (10), in which case ρ_0 and H are simply best-fit parameters to a measured (or modelized) density variation. The scale height does not exceed a very few tens of km for the known planets.

Finally, therefore, the equations describing the fall of a (not too massive) meteorite through a planetary atmosphere reduce to the much simplified (but of course only approximate) form

$$\frac{dy}{dt}=-v\cos\psi, \tag{11}$$

$$\frac{dv}{dt}=-\frac{\tau\rho_0 v^2}{m^{1/3}\rho^{2/3}}e^{-y/H}, \tag{12}$$

where we obtain the very useful

$$\frac{dv}{dy}=\frac{\rho_0 v\tau}{m^{1/3}\rho^{2/3}\cos\psi}e^{-y/H}. \tag{13}$$

And this is where the fun begins; it will be useful to keep in mind certain vital statistics pertaining to the Earth and Jupiter.

The Earth:

atmospheric scale height $H \sim 6.7$ km,

atmospheric density ρ_0 at $y=0 \sim 1.7$ kg/m^3,

atmospheric molecular weight$\sim$30$\times$proton mass,

typical entry velocity$\sim$15 km/s,

$\psi \sim 45°$ on average.

The $y=0$ level corresponds of course to the terrestrial surface; although the actual superficial density is closer to 1 kg/m^3, these parameters represent a good fit to an exponential atmosphere up to an altitude of well over 100 km.

Jupiter:

atmospheric scale height $H \sim 22$ km,

atmospheric density ρ_0 at $y=0 \sim 0.2$ kg/m^3,

atmospheric molecular weight$\sim$2$\times$proton mass,

parabolic entry velocity$\sim$60 km/s.

Note that in the case of Jupiter, the $y=0$ level is merely a convenient reference altitude, where the pressure is equal to 1 bar (10^5 N/m^2); the solid surface of Jupiter (if it exists) is of course buried at depths considerably greater than this. Consequently, *negative* values of y are perfectly legitimate in calculations involving Jupiter—they simply correspond to depths below the reference level. Strictly speaking, the above figures apply to altitudes extending to about 100 km above the 1 bar pressure level; for the phenomena which take place higher up, it is preferable to take a 32 km scale height, with an equivalent "1 bar level" density of about 0.03 kg/m^3.

III. ENERGY LOST BY THE METEORITE

Drag slows the meteorite down; its kinetic energy is transferred to the atmosphere via three distinct paths.

(1) The atmosphere through which the object is moving is heated directly by an adiabatic compression.

(2) The body is heated; its subsequent reradiation also heats the atmosphere, while its surface will lose material.

(3) The material which is lost from the projectile through vaporization and/or melting ultimately mixes its kinetic energy into the atmosphere.

These three contributions are of course not unrelated, since their ultimate source is the descending object's kinetic energy. It is however useful in the spirit of a simplified analysis to separate certain components.

The total rate of change of kinetic energy, the ballistic power lost by the meteorite ${}^{\text{tot}}W$, is given by

$$ {}^{\text{tot}}W = \frac{d}{dt}\left(\frac{mv^2}{2}\right) = mv\,\frac{dv}{dt} + \frac{v^2}{2}\,\frac{dm}{dt} \equiv {}^{\text{drag}}W + {}^{ev}W, $$

where the first term on the right-hand side represents the contribution from a change in velocity for a constant mass, and so can be considered the power source for items (1) and (2) above. From the previous section, we know its value to be

$$ {}^{\text{drag}}W = \tau \rho_a v^3 S. $$

The second term represents the energy contribution to the atmosphere from material ejected by the moving meteorite, and so is the source for item (3) above.

A. Direct atmospheric heating

It is important to note first that meteorite entry velocities can be in the tens of km/s, while the velocity of sound in a gas, $\sim \sqrt{\gamma kT/\mu}$ (γ being the ratio of specific heats of the atmospheric gas), is typically in the hundreds of meters per second range: much of the atmosphere is thus penetrated supersonically. Under such conditions, a shock forms in front of the moving object: in front of the bow shock, the atmosphere is largely undisturbed, while within the shock the gas is compressed and heated. The reason is not hard to fathom: the atmosphere immediately in front of the obstacle must adjust itself so that its local sound speed is higher than the relative velocity of the meteorite and the gas flowing towards it, in order that the atmosphere be able to react to the presence of the object. This implies that the gas is compressed and heated, raising the local sound speed, while the flow velocity is lowered locally.

The compression is adiabatic; however, the transition occurs over such a short distance (on the order of the mean free path of the atmospheric molecules) that it cannot be considered even quasistatic, and so conditions on either side of the shock are controlled essentially just by the continuity equations for mass, momentum, and energy flow, augmented by the perfect gas law. Standard texts such as Anderson,[1] should be consulted for a detailed general account of shock flows; a somewhat simpler analysis, limited to (in the present context) the relevant case where the specific heat ratio γ is equal to the monatomic value of 5/3, can be found in Celnikier.[2]

The important results to note for the present purpose concern the asymptotic behavior of the shock when the flow velocity is several times the local velocity of sound; writing P, ρ, n, T, μ, and v for the pressure, density, particle number density, temperature, molecular mass, and flow velocity, respectively, and distinguishing conditions within and without the shock by the subscripts "in" and "out," respectively, it turns out that

$$ \frac{v_{\text{in}}}{v_{\text{out}}} = \frac{\rho_{\text{out}}}{\rho_{\text{in}}} \to \frac{1}{4}, \tag{14} $$

$$ P_{\text{in}} = \frac{\rho_{\text{in}} k T_{\text{in}}}{\mu_{\text{in}}} = n_{\text{in}} k T_{\text{in}} \to \frac{3\rho_{\text{out}} v_{\text{out}}^2}{4}. \tag{15} $$

v_{out} and ρ_{out} are of course the instantaneous speed of the meteorite v and the local atmospheric density ρ_a, respectively.

Substituting 15 km/s for the flow velocity (it makes no difference whether the body is moving with respect to the gas, or vice versa) and 30 proton masses for the molecular weight, we find from Eq. (15)

$$ \begin{aligned} T_{\text{in}} &\approx \frac{3v_{\text{out}}^2}{4k}\,\frac{\rho_{out}}{\rho_{in}}\,\mu_{in} \\ &= 3\mu_{in} v^2/16k \\ &\approx 10^5 \text{ K in the case of the Earth.} \end{aligned} \tag{16} $$

Thus, a meteorite entering the terrestrial atmosphere will be surrounded by a region of very hot gas; of course, the temperature will drop as the meteorite's speed decreases, but even at a few km/s, the temperature will still be above 1000 K. This is why it is perfectly legitimate to use $\gamma=5/3$: at these temperatures the molecules will be dissociated into atoms, which at the higher speeds will even be ionized. Taking into account dissociation and ionization, the highest temperatures are lowered by a factor of about 4. The meteorite is thus surrounded (at least for a significant part of its trajectory) by a medium at about 25 000 K—which is still pretty hot. This is largely the source of the light emitted as the meteorite descends.

Note that for Jupiter, the mean molecular mass of the undissociated atmosphere is about 15 times lower; however, the entry velocity is about 4 times higher, and so the final conclusions are substantially the same.

These estimates are of course upper limits, for the case where the flow velocity is very much higher than the local sound velocity. Moreover, no account has been taken of various cooling mechanisms which will inevitably be acting, nor of the effect on the surrounding medium of the hot material shorn off the meteoritic surface. Nevertheless, this crude estimate already gives an inkling of what is in store for the meteorite.

B. Meteoritic heating

The hot atmosphere surrounding the descending body will heat its surface, either through the absorption of part of the radiation emitted by the shocked gas, or through frictional interaction with the gas flow.

Now, very roughly, the maximum rate at which kinetic energy can be transferred to the obstacle will be a function of $v_{\text{in}}^2 \times v_{\text{in}} \propto v^3$, while the rate at which radiation energy will be absorbed is a function of $T_{\text{in}}^4 \propto v^8$ (in practice, note that radiation energy is absorbed at a rate which depends on a power of the velocity which lies in the range 5–12, according to the object's shape, see Anderson).[3]

In principle, the total energy transferred is the sum of frictional and radiative contributions; however, since these two terms are very different functions of v, they presumably cross at some limiting value of the velocity, sufficiently far below which the main source of heat will be friction, while at sufficiently high speeds radiation heating will dominate. Frictional heating is relatively inefficient; when radiative heating dominates, the transfer of energy is rather more efficient, since on the order of one-half of the radiation emitted by the shocked gas can illuminate the meteoritic surface.

The speed v_{lim} at which the contributions are equal is the intersection of the two energy functions, and we may locate this crossover point by equating the kinetic power absorbed by the meteorite, W (some fraction f of ${}^{\text{drag}}W$), to the power radiated by the shocked gas towards the object, $S\sigma T_{\text{in}}^4/2$ (where σ is Stefan's constant, and the factor 2 takes rough account of the fact that the shocked gas radiates not only towards the meteorite but also away from it). Thus

$$W = f\,{}^{\text{drag}}W = f\tau\rho_a v^3 S, \tag{17}$$

while the radiative power absorbed by the surface, assuming that the shocked gas is dissociated and ionized, is given by

$$\epsilon\,\frac{S\sigma T_{\text{in}}^4}{2} = \frac{\epsilon S\sigma}{2}\left(\frac{3\mu v^2}{64k}\right)^4, \tag{18}$$

where ϵ is the absorption coefficient of the surface. Equating these two expressions gives us an expression for the limiting velocity $v_{\lim}$ beyond which radiation rapidly becomes the dominant influence

$$v_{\lim} = \left(\frac{2\,f\tau\rho_a}{\epsilon\sigma}\right)^{1/5}\left(\frac{64k}{3\mu}\right)^{4/5}.$$

It might perhaps be worthwhile emphasizing that the equality of Eqs. (17) and (18) is not implied over the entire velocity range, and thus does not impose a particular functional dependence of $f\tau/\epsilon$ on v, but merely fixes the velocity $v_{\lim}$ where the two energy terms make an equal contribution to the power absorbed.

Now, the ratio f/ϵ is likely to be in the range 0.1–0.01, but this particular result is not unduly sensitive to its exact value since it appears raised to the power 1/5. A more important factor is the atmospheric density, through its exponential variation with altitude. Putting $(2\,f\tau/\epsilon)^{0.2} \approx 0.5$, one finds that in the case of the Earth, the limiting velocity works out to about 4 km/s at an altitude of 50 km, rising to a little under 8 km/s at 25 km. In the case of Jupiter, it turns out to be about 0.1 km/s at 1000 km above the reference level, rising to about 40 km/s at 100 km and 60 km/s at 50 km.

We thus see that radiative heating will certainly dominate in the upper parts of the atmosphere for objects entering with at least a parabolic velocity; however, the power actually dissipated at very high altitudes is in fact very small. As the meteorite penetrates the atmosphere, its speed decreases and the limiting velocity rises; a relevant altitude to consider is that at which power dissipation is a maximum, which for a 1 ton nonevaporating body in the case of the Earth [see below, Eq. (22)] turns out to be in the region of 10 km. A re-entering astronaut might justifiably be wary of trusting his life to such a rough and ready analysis; interestingly enough, Anderson,[3] states that radiation heating was in fact slightly less than friction for the Apollo command module returning from the Moon.

Be that as it may, the power absorbed by the meteorite from whatever source will (almost) always be some fraction f of the kinetic power ${}^{\text{drag}}W$ dissipated during the descent; the maximum value of f will be about (0.5×the absorption coefficient of the surface)≈0.2 when radiative heating by the shocked gas dominates, but f is in general somewhat less, and will often be a function of speed.

For our purposes (atmospheric entry of cosmic matter), $f=0.1$ is a reasonable compromise value biased strongly to the higher speeds which favor radiation heating.

One exception to this rule should be noted. It can conceivably happen that as the projectile penetrates to the lower atmosphere, the increased energy dissipation is consumed by increased ionization of the shocked gas (in the case of atmospheres whose atoms can be multiply ionized); in such a case, the temperature of the shocked gas may well stabilize, and if its radiation dominates the meteoritic heating, the power absorbed by the meteorite in this regime will no longer depend on velocity. I shall ignore this possibility, which should not in any case play a significant role for Jupiter.

The heated surface of the meteorite, at temperature T_s, will

(1) radiate energy, essentially at a rate given by Stefan's law $6S\sigma T_s^4$, assuming for simplicity that the entire surface radiates uniformly—the surface area of a cube is six times that of a face;
(2) sublimate and/or melt, depending on the temperature reached.

Thus, even if the meteorite does not break up under the action of the dynamic pressure, it will always gradually lose material.

It is essential to assess the relative importance of various processes, and to do this we shall first assume that the material loss is negligible; this allows one to integrate Eq. (13) trivially, obtaining

$$v = v_e \exp\left\{-\frac{\tau\rho_0 H}{m^{1/3}\rho^{2/3}\cos\psi}\,e^{-y/H}\right\}, \tag{19}$$

where v_e is the entry velocity. Notice that the object's mass appears in this expression only to the power 1/3 (albeit as the argument of an exponential); the qualitative behavior of the descent trajectory is not expected to be a sensitive function of the mass, nor of the way it changes during the flight.

Substituting in Eq. (17), the meteorite absorbs energy at the rate W:

$$W = \left(\frac{m}{\rho}\right)^{2/3} f\tau\rho_0 v_e^3 e^{-y/h} \times \exp\left\{-\frac{3\tau\rho_0 H}{m^{1/3}\rho^{2/3}\cos\psi}\,e^{-y/H}\right\}. \tag{20}$$

This quantity is a product of an exponential which rises with decrease in altitude, and another which falls; consequently, there must be a maximum W_m which is essentially located in the usual way

$$W_m = \frac{fmv_e^3\cos\psi}{3He}. \tag{21}$$

The maximum occurs at an altitude y_m given by

$$y_m = H\ln\frac{3\tau\rho_0 H}{m^{1/3}\rho^{2/3}\cos\psi}. \tag{22}$$

We shall find these parameters useful to simplify notation later on.

It is similarly easy to show that the deceleration dv/dt suffered by the meteorite during its descent goes through a maximum

$$\left[\frac{dv}{dt}\right]_{\max} = \frac{v_e^2\cos\psi}{2He},$$

at an altitude equal to

$$H \ln \frac{2\tau\rho_0 H}{m^{1/3}\rho^{2/3}\cos\psi}. \quad (23)$$

The deceleration is a measure of the mechanical stresses to which the object will be subjected; in this constant mass case, the altitude of maximum deceleration differs from that of maximum power absorption by a fixed (small) constant.

The application of these results in the context of spacecraft re-entry is discussed in Celnikier,[2] as is also an amusing analogy with the way the electron density varies with altitude in an ionosphere.

Using W_m and y_m, the power loss Eq. (20) can be expressed in the very compact parametric form

$$\frac{W}{W_m} = e^{(y_m - y)/H} \exp[1 - e^{(y_m - y)/H}]. \quad (24)$$

Let us continue to ignore possible sublimation and melting processes, and merely assume that thermal equilibrium has been established between the radiation of the heated surface and its absorption of energy; this will give an estimate of the highest temperature $T_{\max}$ to which the meteorite can ever aspire:

$$6S\sigma T_{\max}^4 \approx \frac{fm \cos\psi}{3He} v_e^3,$$

so that

$$T_{\max} \approx m^{1/12}\rho^{1/6} v_e^{3/4} \left(\frac{f \cos\psi}{18\sigma He}\right)^{1/4}.$$

The maximum temperature is clearly a sensitive function only of the entry velocity; even for kg masses (which in fact may not survive to reach y_m, but that is irrelevant for the present illustrative purpose) and an entry velocity of around 15 km/s in the terrestrial atmosphere, the superficial temperature of the body would reach on the order of 10^4 K if the body could eliminate the absorbed energy only by radiation.

However, even an ordinary piece of iron does sublime (albeit poorly at room temperature). Could a meteorite be cooled significantly by the ejection of material from its surface?

This problem can be studied in much the same way as the evaporation of a comet under the influence of solar radiation—see, e.g., Celnikier and Meyer.[4]

Writing the latent heat of vaporization per unit mass of the meteoritic material as L, the power needed to maintain a mass flux F_m (mass per unit surface area per unit time) from the entire surface, ${}^{ev}W$, is given by

$${}^{ev}W = 6SLF_m,$$

the factor 6 again taking into account the fact that the total surface area of a cube (the chosen shape) is six times that of a single face. Thus, assuming that evaporation has not taken away so much material along the trajectory that Eq. (21) for the maximum power has been invalidated, equating power absorbed and power "used" to radiate and to evaporate, one finds

$$W_m = 6S\sigma T^4 + 6SLF_m, \quad (25)$$

Suppose that from the point of view of the meteorite's surface, the vapor remains in equilibrium with it (surely not a very good assumption, but quite adequate for our present purpose which is just to obtain a rough idea of the thermal conditions); the numerical flux of evaporating particles, F_n (number per unit surface per unit time), is then proportional to the saturation vapor pressure P_{sat}, with (see Tabor)[5]

$$F_n \approx n\langle u\rangle/4, \quad (26)$$

$$P_{\text{sat}} = nkT, \quad (27)$$

where

n = number density of evaporated particles,

$$\langle u\rangle = \text{mean thermal velocity of evaporated particles} = \sqrt{\frac{8kT}{\pi\mu_m}}, \quad (28)$$

and μ_m is the molecular mass of the evaporated particles. Consequently the mass flux is given by

$$F_m = \mu_m F_n \approx P_{\text{sat}} \sqrt{\frac{\mu_m}{2\pi kT}}. \quad (29)$$

Consider two very different materials—iron and ice. The latter is very volatile; the former is its antithesis. The saturation vapor pressure of any material increases with temperature in an approximately exponential way (see Tabor);[5] fitting the appropriate exponential law to the values tabulated in Gray,[6] one obtains in a straightforward way

$$P_{\text{sat}} \sim \begin{cases} 7\times 10^{11} \exp(-4.7\times 10^4/T) \ \text{N/m}^2 \text{ for iron} \\ 1.8\times 10^{11} \exp(-5.5\times 10^3/T) \ \text{N/m}^2 \text{ for ice.} \end{cases} \quad (30)$$

Note that although the saturation vapor pressures of different materials are very different, and have a very sensitive dependence on temperature, the latent heats per unit mass (of vaporization and melting) cover a much smaller range, and are much less dependent on temperature; for example,

$$L \approx \begin{cases} 6\times 10^6 \ \text{J/kg for the vaporization of iron} \\ 2\times 10^6 \ \text{J/kg for the vaporization of ice.} \end{cases}$$

This fact is related to the molecular structure of matter.

Finally, substituting Eq. (29) in the power balance equation, Eq. (25), and rearranging, we obtain at the point where energy is being dissipated at a maximum rate

$$\frac{fm^{1/3}\rho^{2/3}\cos\psi}{18He} v_e^3 = \sigma T^4 + \frac{LP_{\text{sat}}}{\sqrt{T}} \sqrt{\frac{\mu_m}{2\pi k}}. \quad (31)$$

The saturation vapor pressure is an exponential function of temperature; consequently, at sufficiently low temperatures, the thermal radiation term, σT^4, dominates the right-hand side of this expression, but at some value of temperature the two terms are equal, and thereafter the second term rapidly becomes, by an overwhelming factor, the more important of the two. The two terms are equal, for the case of iron, at $T \approx 2000$ K, and at $T \approx 200$ K for the case of ice. Now, the value of the left-hand side of the above expression is several times 10^8 even for kg masses (and for finite values of $\cos\psi$ and f), which is very much larger than the value of σT^4 at these temperatures—the radiative term may therefore be safely neglected, since the second term rises rapidly with temperature. Thus, in practice, the power balance equation reduces to

$$\frac{fm^{1/3}\rho^{2/3}\cos\psi}{18He} v_e^3 = \frac{LP_{\text{sat}}}{\sqrt{T}} \sqrt{\frac{\mu_m}{2\pi k}}, \quad (32)$$

where the appropriate expression for P_{sat} is taken from Eq. (30). Equation (32) is solved easily either by graphical means or iteratively. We find immediately that a 1 kg mass of iron entering the Earth's atmosphere will reach a maximum equilibrium temperature of between 3000 and 4000 K for a value of f in the range 0.002–0.2 respectively; the equivalent equilibrium temperatures in the case of ice are ~350 and 500 K. It is amusing to note how different these temperatures are from the nominal tens of thousands of degrees that the meteorite's surface would have reached in the terrestrial atmosphere in the absence of vaporization, even for the case of a relatively nonvolatile material such as iron. One can thereby appreciate how useful and efficient are the ablative shields on the front surfaces of "traditionally" constructed reentering spacecraft—suitable materials have a saturation vapor pressure between that of ice and iron.

These temperatures are of course a rising function of mass and entry velocity; however, mass only appears to the 1/3 power, and the equilibrium temperature is in any case very insensitive to even quite large changes on the left-hand side of Eq. (32), since evaporation serves as a rather good thermostat. One might expect the temperature range calculated to be typical for a wide variety of conditions, but this can be so only up to a point, because the rapid rise of the saturation vapor pressure, Eq. (30), starts to flatten out above a critical temperature; thereafter, evaporation is increasingly less able to brake the temperature rise so that the meteorite is forced to rely on reradiation (a much less efficient process) to eliminate the power absorbed.

Note also that even when vaporization is taken into account, the maximum equilibrium temperatures are above the melting points of both materials; ice melts at 273 K, of course, and iron at ~1500 K. And in effect, the surfaces of meteorites which reach the Earth's surface show signs of melting. In principle, melting could lead to a more rapid wastage of the meteorite, since the latent heat of melting is lower than that for vaporization; however, the melted material can only be carried away by the atmospheric gas flow, whose coupling to the meteoritic surface is far from elementary; in the spirit of simplicity, we shall limit ourselves to vaporization as the only ablative process, keeping in mind, however, that a complete analysis should take into account material run-off due to melting.

IV. MASS LOST BY THE METEORITE

We have seen that, at the point where the meteorite is losing its kinetic energy the fastest, that fraction f which is absorbed by the body is "used" for the most part to evaporate superficial mass. Let us assume that this remains essentially true over most of the descent trajectory. In that case the total mass loss rate dm/dt is given by

$$\frac{dm}{dt}=\frac{f^{\text{drag}}W}{L}=\frac{fmv}{L}\frac{dv}{dt}, \tag{33}$$

whence

$$m/m_e=\exp f\,\frac{v^2-v_e^2}{2L}, \tag{34}$$

where the index e indicates entry values.

With this relation, we can now aspire to a better estimate of the way the descent velocity varies with altitude. The most rigorous way to do this is via a special function $Ei(u)$, which is tabulated for various values of the argument u; this is explained in Ref. 7.

However, I see little virtue in rigor when the fundamental data on which the final result depends are flawed (we shall see below that f/L is a critical parameter whose value determines everything, but about which we really know very little in the particular context of asteroidal and cometary bodies, and certainly not as much as one would like in the case of meteoritic parent bodies); Zahnle,[8] shows one way to analyze the meteor problem in an analytic way (essentially in the context of the Venusian atmosphere), but I shall use here a rather different technique which also circumvents the necessity of finding a table of $Ei(u)$, and allows all the calculations to be carried out sufficiently accurately on a scientific calculator (albeit preferably equipped with a rudimentary graphics capability). Analytical expressions have the supreme advantage that one can see immediately how various quantities scale.

To begin with, we substitute the above mass variation into the differential equation for v, Eq. (13), giving

$$\frac{dv}{dy}=\frac{\rho_0 v\,\tau}{m_e^{1/3}\rho^{2/3}e^{f(v^2-v_e^2)/6L}\cos\psi}\,e^{-y/H}. \tag{35}$$

This equation has no solution in closed form. We can "construct" an approximate solution; however, any such approximation must at the very least satisfy the boundary conditions that $dv/dy\to 0$ in the limits of $v\to 0$ and $v\to\infty$, and of course that v itself tends to 0 and v_e in the limit of $y\to-\infty$ and $+\infty$, respectively. One can invent many ways to do this; the easiest is to simply modify the constant mass solution, Eq. (19), by introducing a multiplicative parameter χ in the exponent

$$v=v_e\exp\left(-\frac{\chi\tau\rho_0 H}{m_e^{1/3}\rho^{2/3}\cos\psi}\,e^{-y/H}\right). \tag{36}$$

To find "the best" value for the parameter, we first substitute this "modified" solution back into Eq. (35), obtaining after a little manipulation

$$\chi=\exp\left\{\frac{fv_e^2}{6L}\,(1-v^2/v_e^2)\right\}. \tag{37}$$

Now, by its construction, the modified solution (36) certainly has the correct asymptotic behavior; of course, it will strictly satisfy Eq. (35) at only one particular altitude, but since the boundary conditions are also satisfied (and since one does not expect unpleasant discontinuities or other exotica), it can be considered a reasonable approximation to the exact solution of the original nonintegratable Eq. (35). We are in some sense "fitting" a plausible parametrized guess. The point at which we force (or "collocate") the solution to be exact is a rather delicate matter; it should be done if possible at a point where the parameters of the falling body are changing particularly rapidly. A reasonable compromise is to collocate at the point where the kinetic energy of the meteorite is changing fastest—in this way we take optimal account of changes in both mass and velocity.

The time rate of change in kinetic energy is given by ${}^{\text{drag}}W$:

$$\begin{aligned}{}^{\text{drag}}W &= \tau\rho_a v^3 S \\ &= \left(\frac{m_e}{\rho}\right)^{2/3} \tau\rho_0 v_e^3 e^{-y/H} \\ &\quad\times\exp\left\{-\frac{3\tau\rho_0 H\chi}{m_e^{1/3}\rho^{2/3}\cos\psi}e^{-y/H}\right\} \\ &\quad\times\exp\left\{\frac{fv_e^2}{3L}\left[\exp\left(-\frac{2\chi\rho_0\tau H}{m_e^{1/3}\rho^{2/3}\cos\psi}e^{-y/H}\right)-1\right]\right\}.\end{aligned} \tag{38}$$

This function is a product of an exponential which rises as the projectile penetrates the atmosphere, and two exponentials which decrease; consequently there must be a maximum along the trajectory. To locate its position, it is advantageous to simplify the appearance of the function by writing it in parametric form using the quantities W_m and y_m of Sec. III B, giving

$$\begin{aligned}\frac{{}^{\text{drag}}W}{W_m} &= \frac{e^{(y_m-y)/H}}{f}\exp\left\{1-\chi e^{(y_m-y)/H}+\frac{fv_e^2}{3L}\right. \\ &\quad\left.\times\left[\exp\left(-\frac{2\chi e^{(y_m-y)/H}}{3}\right)-1\right]\right\} \\ &\equiv \frac{u}{f}\exp\left\{1-\chi u+\frac{fv_e^2}{3L}\left[e^{-2\chi u/3}-1\right]\right\},\end{aligned} \tag{39}$$

where

$$u=e^{(y_m-y)/H}.$$

In the form of Eq. (39), the time rate of change of the bolide's kinetic energy is really much easier to manipulate. Differentiating with respect to u and equating to zero, we find immediately that the maximum (the point where I have chosen to collocate) occurs at u_{col}, where

$$\left(\chi u_{\text{col}}+\frac{2\,fv_e^2}{9L}\chi u_{\text{col}}e^{-2\chi u_{\text{col}}/3}\right)=1. \tag{40}$$

This transcendental equation is easily solved, iteratively or graphically. Note that the value of χu_{col} (and so also of χ and u_{col} individually) is independent of the mass and density of the projectile, but depends only on quantities related to the entry trajectory, the latent heat of vaporization L of the material, and on the fraction f of the bolide's kinetic energy which goes into evaporating. Substituting the solution from this equation into Eq. (36) gives us the velocity at the collocation point

$$\frac{v_{\text{col}}}{v_e}=\exp\left(-\frac{\chi u_{\text{col}}}{3}\right),$$

and so allows the value of χ to be calculated trivially from Eq. (37).

For example, the value of χ for a piece of iron entering the Earth's atmosphere (assuming $f=0.2$) is about 1.4; it rises to about 1.6 for the same piece of iron entering the jovian atmosphere. Of course, the smaller the value of f, the closer we are to the constant mass case, and so the closer χ is to unity.

Note that the heuristic technique employed here to obtain an approximate analytical solution to an equation which has no solution in closed form is very powerful, but its success does hinge on some *a priori* knowledge of the way the solution should behave asymptotically, and of course on an absence of discontinuities; the reader is encouraged to consult Action and Squire,[9] for a general survey of the method, while some examples related to stellar structure are given in Celnikier.[10]

V. TOTAL ENERGY TRANSFERRED TO THE ATMOSPHERE

In Sec. III A, we saw how the immediate environment of the projectile was affected. It is now opportune to reflect in a little more detail upon the total power transferred to the atmosphere by the descending object, since ultimately that is what one might expect to be able to measure.

The atmosphere is energized by two (related) processes.

Drag: the change in speed of the descending body leads to a transfer of energy into the bow shock, and a transfer of a (lesser) amount of energy to the body, whose surface reacts by vaporizing. Even if the radiation from the shock dominates, the former contribution is rather larger than the latter, so that drag will essentially transfer to the atmosphere the power ${}^{\text{drag}}W$.

Vaporized material: matter which is ejected at the instantaneous speed of the meteorite mixes with the ambient atmosphere and transfers its kinetic energy to it. This corresponds to the power ${}^{ev}W$:

$${}^{ev}W=\frac{v^2}{2}\frac{dm}{dt}={}^{\text{drag}}W\frac{fv^2}{2L}$$

using Eq. (33).

Thus ultimately, the atmosphere receives a total power ${}^{\text{tot}}W$:

$${}^{\text{tot}}W={}^{\text{drag}}W\left(1+\frac{fv^2}{2L}\right)\approx{}^{\text{drag}}W\frac{fv^2}{2L},$$

since over much of the trajectories resulting from planetary encounters and for typical materials, $fv^2/2L$ is very much greater than 1. Consequently (taking into account the mass variation along the descent)

$$\begin{aligned}{}^{\text{tot}}W &= \left(\frac{m_e}{\rho}\right)^{2/3}\frac{f\tau\rho_0 v_e^5}{2L}e^{-y/H} \\ &\quad\times\exp\left\{-\frac{5\tau\rho_0 H\chi}{m_e^{1/3}\rho^{2/3}\cos\psi}e^{-y/H}\right\} \\ &\quad\times\exp\left\{\frac{fv_e^2}{3L}\left[\exp\left(-\frac{2\chi\rho_0\tau H}{m_e^{1/3}\rho^{2/3}\cos\psi}e^{-y/H}\right)-1\right]\right\}.\end{aligned} \tag{41}$$

As one might expect, this function is similar to Eq. (38); it has a maximum which is found in much the same way. As before, we simplify the appearance of the function by writing it in parametric form using the quantities W_m and y_m of Sec. III B, giving

$$\frac{{}^{\text{tot}}W}{W_m}=\frac{v_e^2}{2L}\,e^{(y_m-y)/H}\exp\left\{1-\frac{5\chi}{3}\,e^{(y_m-y)/H}+\frac{fv_e^2}{3L}\right.$$
$$\left.\times\left[\exp\left(-\frac{2\chi e^{(y_m-y)/H}}{3}\right)-1\right]\right\}$$
$$\equiv u\,\frac{v_e^2}{2L}\exp\left\{1-\frac{5\chi u}{3}+\frac{fv_e^2}{3L}\,[e^{-2\chi u/3}-1]\right\}, \quad (42)$$

with as before

$$u=e^{(y_m-y)/H}.$$

Differentiating and equating to zero, we find immediately that the maximum occurs at $u_{\max}$, where

$$u_{\max}\left(\frac{5\chi}{3}+\frac{2\,f\chi v_e^2}{9L}\,e^{-2\chi u_{\max}/3}\right)=1. \quad (43)$$

This transcendental equation is as easy to solve as Eq. (40); indeed, to all intents and purposes, it is identical to it, since in practice $5/3\ll 2\,fv^2/9L$. Finally, therefore, the altitude at which the power dissipation is a maximum, $y_{\max}$, taking into account mass loss from the descending projectile is given by

$$y_{\max}=H\,\ln\frac{3\tau\rho_0 H}{m_e^{1/3}\rho^{2/3}u_{\max}\cos\psi}. \quad (44)$$

At this altitude, the power dissipation to the atmosphere is given by

$$W_{\max}=\frac{fm_e u_{\max}v_e^5\cos\psi}{6HL}\,e^{-5\chi u_{\max}/3}$$
$$\times\exp\left\{\frac{fv_e^2}{3L}\,(e^{-2\chi u_{\max}/3}-1)\right\}. \quad (45)$$

With these quantities, the power delivered to the atmosphere, Eq. (41), is conveniently written in the form

$$\frac{W}{W_{\max}}=\frac{u}{u_{\max}}\exp\left\{\frac{5a}{3}\left(1-\frac{u}{u_{\max}}\right)\right\}\exp\left\{-\frac{fv_e^2e^{-2a/3}}{3L}\right.$$
$$\left.\times\left[1-\exp\frac{2a}{3}\left(1-\frac{u}{u_{\max}}\right)\right]\right\}, \quad (46)$$

where

$$a=\chi u_{\max}.$$

Beyond the maximum, the power delivered to the atmosphere drops precipitously. Defining a "terminal" altitude as one where the power delivered to the atmosphere has dropped to some sufficiently small fraction, say 10^{-5}, of the maximum, the corresponding value of the ratio $u/u_{\max}$, which is in fact equal to $\exp(y_{\max}-y)/H$, can be found from Eq. (46). As for all the numerical applications here, this can be done graphically or iteratively; in the latter case, however, since the equation has some peculiarities, the easiest and most efficient technique turns out to be a simple "stepping" procedure, where the value of $u/u_{\max}$ is simply increased from 1 by suitably arranged steps until the desired value of $W/W_{\max}$ is reached.

Another useful quantity is the altitude of maximum deceleration, easily found to be

$$H\,\ln\frac{2\tau\rho_0 H\chi}{m_e^{1/3}\rho^{2/3}\cos\psi}.$$

Comparing this expression with the altitude of maximum power transfer to the atmosphere, $y_{\max}$, one notes that there is no *a priori* reason why they should be similar, in contrast to the constant mass case [compare with Eqs. (22) and (23)].

Finally, a useful "figure of merit" definable at each point on the descent trajectory is the ratio of the instantaneous kinetic energy of the projectile to the instantaneous rate at which energy is being dumped into the atmosphere: this gives an idea of the time scale t_{end} for which the process could continue from that point if the conditions remained the same. One finds

$$t_{\text{end}}=\frac{mv^2/2}{{}^{\text{tot}}W}=\frac{m^{1/3}\rho^{2/3}L}{f\tau\rho_a v^3}. \quad (47)$$

Clearly, this time scale decreases as the projectile penetrates the atmosphere; at the altitude $y_{\max}$ where power is being transferred maximally, the time left to "terminate" from the maximum, t_{term}, is given by

$$t_{\text{term}}\equiv{}^{\max}t_{\text{end}}$$
$$=\frac{3HL}{fu_{\max}v_e^3\cos\psi}\,e^{\chi u_{\max}}\exp\left\{\frac{fv_e^2}{6L}\,[e^{-2\chi u_{\max}/3}-1]\right\}. \quad (48)$$

One notes that this figure is completely independent of mass; however, it is rather sensitive to the value of f.

VI. SOME COMMENTS

The technique used here to find an approximate solution to what is essentially a nonintegratable problem allows one to judge immediately the sensitivity of the meteoritic (or whatever) trajectory to various parameters.

Consider for the sake of argument a 500 kg iron sphere, entering the terrestrial and jovian atmospheres at 45 degrees to the local vertical. Using the above relations, it is easy to evaluate the salient features of the trajectory; some figures are given in Table I.

A number of interesting points emerge.

As one would expect from the analytical expressions, the results are critically dependent on the value of f (or rather, on the ratio f/L), particularly so for the jovian conditions.

The descent trajectory is thus a sensitive function of f, v_e, and L. However, while the entry velocity can be known relatively well, the latent heat of vaporization is in general not known to better than a few hundred percent since the exact composition of the objects is poorly understood, and the value of f is known to at best an order of magnitude, since it depends on the surface of the object ··· of which we can only surmise the nature. A conventional value of 0.05 is often quoted for low velocities: it has little practical justification. However, choosing the wrong value of f can completely alter our picture of meteoritic penetration; we can see from Table I that the point of maximum power transfer is moved by nearly 60 km (3 scale heights) in the jovian case over the f range considered. Indeed, we usually know neither the mass nor the density of entering bodies; the altitude of maximum power transfer (important for assessing how the atmosphere itself will be affected) depends on $\rho^{2/3}m^{1/3}u_{\max}$, and so quite plausible swings in the density and mass easily accommodate the changes in $u_{\max}$ produced by changes in f. Similarly, the value of the power dissipation itself (which one might expect to observe and measure) depends on both m_e and $u_{\max}$ (but not, interestingly enough, on the density).

Table I. The salient parameters describing the descent of a 500 kg iron sphere through the terrestrial and jovian-type atmospheres, as parametrized in this paper.

Planet	Earth			Jupiter		
v_e	15 km/s			60 km/s		
f	0.01	0.1	0.2	0.01	0.1	0.2
χ	1.03	1.2	1.4	1.3	1.6	1.6
u_{max}	0.6	0.4	0.2	0.3	4×10^{-2}	2×10^{-2}
W_{max}	2×10^{9}	1.4×10^{10}	1.9×10^{10}	3.3×10^{11}	5.4×10^{11}	5.6×10^{11}
y_{max}	17	21	23	52	93	108
v_{max}	12	13	13	53	59	59
m_{max}	470	311	240	260	129	120
y_{term}	−1	0.7	1.9	−16	24	44
t_{term}	15	1.9	1	1	0.4	0.4
v_{term}	0.7	0.9	1	4	35	48
m_{term}	415	77	12	25	2×10^{-6}	2×10^{-7}

Note: I have assumed that the drag parameter τ takes the value 0.5, and the terminal altitude is taken to be that where the power delivered to the atmosphere has fallen to 10^{-5} of its maximum value. Velocities are in km/s, distances in km above the reference level, power in watts, and masses in kg; the time t_{term} is in s.

Another interesting feature is the distance between the altitude of maximum power transfer and the terminal altitude: it hovers at a very few scale heights, *whatever* the value of f, entry velocity, and atmosphere. Of course, its exact value depends on the precise criterion used to define y_{term}, but it is clear that for a given choice, the variation of this distance will not be very great. Moreover, since Eq. (46) is independent of the mass of the falling object, this distance is also independent of mass ⋯ at least to within the approximations of this paper: certainly, some variation with mass is to be expected in practice, since different masses penetrate to different depths and so encounter in practice different atmospheric structures, but the effect should not be greater than the variation in the local scale height. All this has an interesting consequence: the time to extinction from the maximum turns out in all cases to be on the order of a very few seconds, depending of course on the entry velocity.

A perfectly equivalent way to see this is from the value of t_{term}, Eq. (48); as already noted, this quantity is effectively not a function of mass, but does depend explicitly on the local atmospheric scale height. Note that the numerical results are not necessarily the same as those based on the time to extinction, since the criteria are quite different and the conditions at the maximum used to evaluate t_{term} are not representative of the subsequent flight.

Comparing the published results of complex numerical models with simplified analytical ones is generally a tricky business, since the exact numbers used in the former are rarely available in print—noteworthy exceptions are Zahnle,[8] Chyba *et al.*,[11] Hills and Goda.[12] Another reasonably well-documented paper, which we shall use here, is Baldwin and Sheaffer,[13] where the fall of meteorites of various compositions was traced numerically through the terrestrial atmosphere, taking very detailed account of the interaction of the atmosphere and the meteoritic surface. A material called "bronzite" furnishes a fairly "clear" example for comparison; its density is 3600 kg/m^3, with a latent heat of vaporization of 8.6×10^6 J/kg. All calculations were done for an entry angle of 52°, the object being supposed spherical corresponding to a value of $\tau\sim0.7$; tabulated values were used for the atmospheric structure.

The first quantity of interest is the mass surviving at ground level for various entry speeds. We saw in Sec. III B that one should distinguish between two extreme regimes: relatively low speeds (say less than several km/s) where atmospheric friction is the essential heat source, and high speeds, where radiation from the shock dominates. For the former case, the value of f (the fraction of the incident kinetic energy used to ablate material) is conventionally taken to be ~0.05; it will rise gradually as the speed rises, but in any case cannot much exceed 0.2 so that 0.1 is probably a reasonable compromise value for an entire trajectory.

At 20 km/s, the analytical results of this paper (with $f=0.1$) suggest that only a fraction ~0.1 of a 1000 kg bolide reaches the terrestrial surface, while at 40 km/s ($f=0.1$) the fraction drops to $\sim10^{-4}$. From Fig. 1 of Baldwin and Sheaffer,[13] one finds a few times 0.1 and 10^{-4} for these numbers. Iron has the same thermal constants, but the higher density of 7900 kg/m^3. One can see immediately from Eq. (36) that the higher the density, the more rapidly the superficial velocity approaches zero, and therefore [Eq. (34)] the smaller the fraction of the mass surviving at ground level. Note that this is counterintuitive and one would expect a larger fraction of a denser object to survive; however, the numerical analysis confirms the analytical result. In short, only a vanishingly small fraction of the fastest bodies can reach the surface.

The rate of ablation constitutes another interesting point of comparison. This is given by Eq. (33); taking, as in Baldwin and Sheaffer,[13] a 500 kg bronzite projectile entering with a speed of 14.2 km/s ($f=0.1$) inclined at 52° to the vertical, I find that the maximum ablation rate occurs at just 24 km, where the object is moving at just over 12 km/s and is losing mass at close to 100 kg/s. Figure 15 of Baldwin and Sheaffer,[13] shows a maximum at 25 km, with a loss rate of 100 kg/s; the speed there is a little over 10 km/s.

The agreement seems quite satisfactory; one can clearly obtain useful results from the simple analytical relations of the present paper, and with much less effort than is required to build a full scale simulation. Of course, the reliability of the answers (be it from the analytical relations or the simulations) depends on the reliability of the basic data ⋯ which is poor; indeed, the differences from the numerical model are much smaller than differences arising from plausible changes in the fundamental data.

VII. METEORITIC BREAKUP

In all the analysis, I have so far assumed that the projectile is perfectly rigid; mass is supposedly removed gradually.

This is hardly likely to be true beyond some point on the trajectory; melting due to the high temperatures reached and breakup due to the aerodynamic (sometimes called "ram") pressure could completely alter the conclusions.

A. Melting

The maximum superficial temperature attained by a body which is evaporating, but whose mass loss is a negligible fraction of its mass, was estimated in Sec. III B. The results obtained since then can be applied in much the same way to obtain an expression for the surface temperature at any point along the trajectory of a body losing a significant fraction of its mass; the power absorbed by the meteorite is still given by Eq. (17), while the expression for the velocity should be that in which account has been taken of the mass loss, Eq. (36), so that

$$W=f\tau\rho_a S v_e^3 \exp\left(-\frac{3\chi\tau\rho_0 H}{m_e^{1/3}\rho^{2/3}\cos\psi}e^{-y/H}\right).$$

The power used to evaporate material is, as before

$$6SLP_{\text{sat}}\sqrt{\frac{\mu_m}{2\pi kT}},$$

where the saturation vapor pressure P_{sat} is given by Eq. (30), and the factor 6 accounts as before for evaporation from all six faces of the falling cube. Finally therefore

$$f\tau\rho_0 S v_e^3 e^{-y/H} \exp\left(-\frac{3\chi\tau\rho_0 H}{m_e^{1/3}\rho^{2/3}\cos\psi}e^{-y/H}\right)$$
$$=6LP_{\text{sat}}\sqrt{\frac{\mu_m}{2\pi kT}}.$$

Each value of y corresponds to a simple transcendental equation for the temperature; the easiest way to avoid some tedious calculation is simply to plot the left- and right-hand sides of the above relation, and just read off the temperature at each altitude—great precision is not required since we only need at this stage an idea of the solution. Taking for the sake of argument a 500 kg piece of iron entering the terrestrial atmosphere at 15 km/s (and so with $f=0.05$, the conventional value), one finds that already at an altitude of 100 km, the surface temperature exceeds several thousand degrees, reaching a maximum of about 7000; falling through the jovian atmosphere (with an initial speed of 60 km/s), the iron would again have reached several thousand degrees at 500 km above the 1 bar level, and about 8000 at maximum. The temperatures are roughly an order of magnitude lower for a piece of ice.

Such results are of course only indicative, because at these temperatures the saturation vapor pressure relation used is at best very approximate; however, it is clear that surface melting must be a significant feature over a considerable fraction of the descent trajectory.

The first consequence could be a much higher rate of erosion, since the latent heat of melting can be an order of magnitude lower than that of vaporization. Indeed, changing the latent heat by as little as a factor of 5 changes the terminal altitude by between one and two scale heights (depending on trajectory details)—using an inappropriate value for the latent heat has consequences as catastrophic, and is as easy, as using a faulty value of f. Although we usually know the latent heats of a pure material, we know very little about the thermal properties of the composite structures that make up asteroids and comets; while laboratory measurements of the properties of meteorites have been done (see, e.g., Ceplecha),[14] one should bear in mind that these are merely the surviving, and to my mind hardly representative, fragments of much larger parent bodies. In practice, the ratio f/L is sometimes referred to as the "ablation coefficient;" however, this nomenclature does not make its value any more reliable for the bodies in which we are really interested.

Melting can be catastrophic if there is an efficient removal mechanism; however, even if there is not, melting could proceed right through the object, fatally weakening its coherence and causing it to break up well before it has been eroded away.

For global melting to take place, the internal temperature must rise sufficiently. Consider an analogous, but much simpler problem: a rod of mass m and length x is heated at one end by a power W incident on one face. As a consequence.

(1) There is a temperature gradient $\Delta T/\Delta x$, such that

$$\frac{W}{S}=\lambda\frac{\Delta T}{\Delta x},$$

where λ is the thermal conductivity.

(2) The characteristic temperature of the mass will rise at the rate $\Delta T/\Delta t$:

$$\frac{W}{cm}=\frac{\Delta T}{\Delta t},$$

where c is the specific heat of the material.

Thus:

$$\frac{\Delta T}{\Delta t}=\frac{S\lambda}{cm}\frac{\Delta T}{\Delta x}.$$

Now, over the depth of the meteorite, evaporation maintains a maximum temperature difference of several thousand degrees. For metals, the thermal conductivity is several tens of W/m/K, while the specific heat is several hundred J/kg/K, so that λ/c is of the order of 0.1. Finally, therefore, for masses larger than several kg, the temperature rise is less than about a degree per second. The estimate is crude, but it suffices to show that the characteristic internal temperature of even a metallic meteorite rises so slowly in spite of the imposed heat flux that global melting is hardly an eventuality for which it is worthwhile carrying out a detailed calculation.

Meteorites of any significant mass will not fall apart through global melting.

B. Mechanical deformation and fragmentation

The drag force responsible for the heating process also represents a pressure difference across the object, and one expects therefore mechanical stresses to be set up. In essence, the deforming force is given by Eq. (8); once its value exceeds some critical number, one can expect pieces to fly off the falling object. The problem is not only that we have little idea of what this critical point might be; we also have no idea by what process the object will disintegrate. A heterogeneous body such as a meteorite may simply fragment along fault lines as soon as the imposed shear becomes too large (this is known to have happened to meteorites penetrating the terrestrial atmosphere); it may shatter into a multitude of minute pieces when the deceleration becomes too high; in the unlikely event of a perfectly homogeneous body, it may change its shape plastically until it becomes so fragile somewhere that pieces break off. The former two possibilities, while realistic, cannot be assessed in any serious way since we do not possess the fundamental data for realistic objects, and indeed the models which have been calculated, in spite of their apparent completeness and numerical rigor (see, e.g., Baldwin and Sheaffer,[13] and Hills and Goda)[12] are based on very particular assumptions of breakup; the latter possibility, plastic deformation, can be evaluated, but is so unrealistic that one wonders whether the effort is worthwhile: composite materials will surely not flow mechanically, and in any case falling objects will tumble, so that deformations cannot be maintained for even short intervals of time.

Nevertheless, suppose that at some point during the descent, the aerodynamic pressure has exceeded the elastic limit for the material; since the force acts on the leading edge, it will tend to "squeeze" together the opposing sides of the object, which will react by becoming flatter and broader. Very crudely, half the mass will be accelerated by the force $\tau S \rho_a v^2$ with respect to the other half; one can thus define at each point along the trajectory a "time scale" $t_{\rm crush}$ needed to force the front-half over the space occupied by the backhalf, so that in the case of our cube of side r (assuming that forces and faces are well aligned and that there is no rotation):

$$\tau S \rho_a v^2 \frac{t_{\rm crush}^2}{2} \frac{2}{m} \approx \frac{r}{2},$$

whence

$$v t_{\rm crush} \approx \frac{m^{1/3} \rho^{1/6} e^{y/2H}}{\sqrt{2\tau\rho_0}} = \Delta d_{\rm crush}.$$

Now, at the instantaneous speed v, $v t_{\rm crush}$ is a measure of the distance $\Delta d_{\rm crush}$ farther traveled by the object before being "flattened," i.e., before something nasty happens to it. The distance $d_{\rm crush}$ corresponds to an altitude change $\Delta y_{\rm crush}$ required to crush the object

$$\Delta y_{\rm crush} = \Delta d_{\rm crush} \cos \psi \approx \frac{m_e^{1/3} \rho^{1/6} \cos \psi}{\sqrt{2\tau\rho_0}} e^{y/2H} \times \exp\left\{ \frac{f v_e^2}{6L} \left[\exp\left(- \frac{2\chi\rho_0 \tau H e^{-y/H}}{m_e^{1/3} \rho^{2/3} \cos \psi} \right) - 1 \right] \right\}. \quad (49)$$

This reasoning is only valid (inasmuch as it is at all valid) for values of $\Delta y_{\rm crush}$ such that atmospheric conditions change little, i.e., over a fraction of the scale height. We can then claim that something special has happened to the falling object at around an altitude $y_{\rm crush}$ in the range $\Delta y_{\rm crush}$.

A number of points are worth noting.

(1) $\Delta y_{\rm crush}$ is a (slowly) rising function of density. Consequently, all other parameters being equal, a denser object will be "crushed" (in the sense outlined here) at a lower altitude than a less dense one. However, this conclusion should be tempered by the knowledge that denser objects are eroded faster—there may not be much left to crush.

(2) $\Delta y_{\rm crush}$ is a rising function of cos ψ. Therefore, the more inclined the incoming trajectory of a given mass, the higher up it will be crushed. This is perhaps counterintuitive; it is in fact a projection effect, since by fixing $\Delta y_{\rm crush}$, we are in fact allowing objects on inclined trajectories more time to be crushed.

(3) The full significance of this "crushing distance" can only be appreciated when compared to the competing "erosion distance" $\Delta y_{\rm erosion}$ defined using the time $t_{\rm end}$ Eq. (47)

$$\Delta y_{\rm erosion} = v t_{\rm end} \cos \psi = \frac{m^{1/3} \rho^{2/3} L}{f \tau \rho_a v^2}.$$

This leads, after a little manipulation, to

$$\frac{\Delta y_{\rm crush}}{\Delta y_{\rm erosion}} = \frac{v_e^2 f}{L} \sqrt{\frac{2\tau\rho_0}{\rho}} e^{-y/2H} \times \exp\left\{ - \frac{2\tau\rho_0 H \chi}{m_e^{1/3} \rho^{2/3} \cos \psi} e^{-y/H} \right\}. \quad (50)$$

Mechanical destruction, as opposed to erosion, plays a significant role when this ratio is much less than 1. Now *in the case of the Earth*, $f v_e^2/L$ usually falls in the range 1–10, except for the highest speeds, while $\sqrt{2\tau\rho_0/\rho} \approx 0.1$ except for the lowest densities. Therefore, mechanical destruction will effectively be favored over erosion, once the elastic limit of the material has been exceeded, except for extremely rapid and low dense bodies.

In the case of Jupiter, the lowest possible value for the entry speed is sufficiently higher for the situation to be not so clear cut; in fact, crushing (in the sense described here) and erosion might well be processes of comparable importance.

(4) A final instructive and easily calculated parameter is the difference between the altitude at which crushing is initiated, $y_{\rm crush}$, and the altitude at which the power transfer to the atmosphere is maximal, $y_{\max}$; from Eqs. (49) and (44), one obtains (almost) immediately

$$y_{\rm crush} - y_{\max} = H \ln \frac{2\Delta y_{\rm crush}^2 u_{\max}}{3 H m_e^{1/3}} - \frac{f}{3L} (v_{\rm crush}^2 - v_e^2), \quad (51)$$

where $v_{\rm crush}$ is the speed when crushing begins. Depending on the conditions, this difference can be positive or negative; if negative, crushing occurs beyond the point where the atmosphere has been maximally perturbed. Very roughly, the sign depends on the initial speed; the higher v_e the lower $u_{\max}$ and so the closer $v_{\rm crush}$ is to v_e—at sufficiently high speeds, $y_{\rm crush} - y_{\max}$ is always negative. Thus, for example, objects entering the terrestrial atmosphere at a low speed will be crushed well before being maximally eroded, while the opposite is true at very high speeds. On the other hand, since entry speeds into the jovian atmosphere are always very high, one expects crushing to occur close to and, for sufficiently massive objects, beyond the altitude of maximum power transfer.

The effect on the atmosphere of crushing the meteorite in this way is not clear. In one scenario, the phenomenon is likened to an "explosion," the residual kinetic energy of the meteorite being supposedly released to the atmosphere on a shorter time scale than if ablation had continued. However, energy will be transferred rapidly to the atmosphere only if crushing has instantly reduced the remaining mass to a near gaseous state; if (as seems intuitively more likely) the result is merely the production of a set of finite fragments, say pebble-sized, the energy transfer will occur on the erosion time scale of each fragment which is relatively long and in the final stages is essentially independent of mass [see Eqs. (47) and (48)]. This latter conclusion is confirmed by the numerical simulation of Baldwin and Sheaffer,[13] where meteoritic breakup in the sense of fragmentation was included in the simulation: compared to a rigid mass, the "terminal altitude" is higher, but the time scale for a significant energy transfer to the atmosphere is quite comparable.

This is the essence of a model presented in Chyba *et al.*,[11] where moreover the braking effect of the increasingly deformed projectile was taken into account through a numerical procedure. The model of course assumed that the gradually squashed body remained permanently with its large face facing the direction of motion—it is far from clear to me that such a hypothesis is justified, but its effect must be to raise altitudes as compared to the results of the above "hand-waving" approach. Nevertheless, in spite of the radically different techniques used, the conclusions are substantially similar. For example, taking a limiting value for Δy_{crush} of 2 km gives the following.

(1) An iron mass (5.6×10^8 kg, 7900 kg/m^3, $L=8\times10^6$ J/kg, entering with a speed of 15 km/s at an angle of 45°) just about reaches the terrestrial surface without being crushed.
(2) A "long period comet" (5×10^7 kg, 1000 kg/m^3, $L=2.5\times10^6$ J/kg, entering with a speed of 50 km/s at an angle of 45°) will find itself in a "delicate situation" at an altitude of about 20 km.

The corresponding values in Chyba *et al.*[11] (see Fig. 1 in that paper) are 0 and 30 km, respectively.

Note however that in the case of the long period comet, the altitude at which power is being transferred maximally by a nonfragmenting body is about 25 km (assuming $f=0.1$), the terminal altitude falling at about 4 km; thus, the altitude at which the atmosphere is perturbed maximally by this type of object is hardly altered by fragmentation; however, as a consequence of mechanical failure, the terminal altitude is somewhat higher, and so the power transferred to the atmosphere could be greater, depending on just what is left (see above).

The paper of Chyba *et al.*,[11] also considers the effect of entry angle on a "stony asteroid" (5.6×10^8 kg, 3500 kg/m^3, $L=8\times10^6$ J/kg, entering with a speed of 15 km/s); at $\psi=75°$, 60°, and 0°, "crushing altitudes" work out to be 15, 7, and 2 km, respectively, while the corresponding figures in that paper are 15, 12, and 6 km.

While an exact comparison is virtually impossible, since the criteria are so different, the agreement is quite satisfying, and certainly within the intrinsic uncertainties of this type of model.

VIII. FINAL COMMENTS

I have shown in this paper that the fall of a meteorite through a planetary atmosphere, taking into account mass erosion and a kind of fragmentation process, is perfectly amenable to a simple analytical treatment, highlighting the sensitivity of the results to just the parameters whose values are poorly known.

The detailed quantitative observation of a fall could, however, allow some parameters to be determined. One can see from Eq. (48) that the time to extinction t_{term} is a function only of the following.

(1) Atmospheric parameters, in principle knowable for a given planet.
(2) The entry trajectory parameters v_e and ψ, in principle knowable if the object were followed before impact.
(3) The ablation coefficient f/L, the only unknown quantity.

Of course, once the ablation coefficient has been determined, one can make a good guess at the value of L, and thus identify of which material the object is made; moreover, other parameters of interest follow:

(1) m_e from Eq. (45);
(2) ρ from Eq. (44), since τ is reasonably well determined.

The impact with Jupiter of comet Shoemaker–Levy could provide a textbook case ··· providing that we can separate the light emitted during the descent from the effect of the subsequent atmospheric "fireball." For this we must await the full data from the Galileo spacecraft, the only instrument which was able to observe the impact "face-on," since it occurred on the far side of the planet with respect to the Earth.

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