equilibrium is found outside the vertical diameter of the circle. This symmetry change is spontaneous in the sense that the symmetry of the environment of the system is not modified as Ω goes through the value Ω_c. In the same way, the time reversal symmetry of an Ising magnet is spontaneously broken at the magnetic transition, as the temperature T is lowered below some critical value T_c.

(2) We check here the general property[5] that the solution of a symmetrical problem is symmetrical only if it is unique. For $\Omega < \Omega_c$, stable equilibrium is found only for $\theta = 0$ and is symmetrical. For $\Omega > \Omega_c$, it is found for two different positions $\theta = \pm\theta_0$ and is nonsymmetrical. The two solutions $\theta = \pm\theta_0$ are similar to the two domains of opposite magnetization of an Ising magnet below T_c, they are symmetry related.

(3) The bifurcation found at $\Omega = \Omega_c$ is a consequence of the softening of the oscillation of M around the equilibrium position $\theta = 0$ as Ω increases. The instability is similar to a displacive[6] phase transition.

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Weighing the Earth with a sextant

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This article presents a simple, but accurate, method for determining the distance of the Moon using a cheap sextant. This result can then be used to obtain the size of the Moon and the mass of the Earth.

The distance of the Moon has played an important role in the history of science. It was the only cosmic distance which the ancient Greeks were able to determine with any kind of precision, while Tycho Brahe, by showing that comets were certainly more distant than the Moon, contributed to the demise of the Aristotelian vision of the universe.

Cosmic distance scales are based on one of two essential methods. Simple triangulation using the largest available base line provides a scale for nearby objects. The Earth's surface furnishes a base line for the Moon and certain asteroids (although to a large extent laser and radar ranging have replaced this method as far as the solar system is concerned), while the annual movement of the Earth itself around the Sun provides a suitable base line for the nearer stars. The distances of distant stars and galaxies are obtained indirectly, using essentially a measure of their apparent brightness coupled to some more or less plausible and more or less well-verified set of hypotheses concerning their intrinsic brightness.

Geometrical methods are inherently more reliable if at all applicable, since no suppositions concerning the nature of the object enter into the procedure. However, they suffer from two drawbacks. In the first place, a large base line is needed, and in the second directions must be determined with high precision.

The Earth furnishes an excellent base line of variable length: during the course of 12 h an observer is carried through a distance of 12 000 km: over such a base line, the direction in space of a fixed object at the distance of the Moon changes by about one degree. This fact was already recognized by Tycho Brahe; its power is that one person can, in principle, carry out all the measurements and it is not necessary to set up a time synchronized team of observers at opposite ends of the globe. The same principle is of course applied to measurements of stellar parallaxes at opposite ends of the Earth's orbit.

While one degree is not an impossibly small angle to measure, it is also by no means trivial with simple instruments—the mountings of most small amateur telescopes are really quite inadequate (in spite of the makers' claims) and cannot be used to obtain a spatial direction to this precision without considerable effort. The problem is compounded by the fact that the Moon is not stationary: its orbital motion is in the same direction as the rotation of the Earth and so the apparent parallax is actually smaller than it should be; moreover, measurements cannot in practice be spread over 12 h and are rarely made at the equator, so that the effective base line is much smaller than 12 000 km.

The mariners's sextant is a rather accurate device. Professional instruments are very expensive, but it has for some time been possible to obtain cheap plastic models which, in spite of their apparent simplicity, are quite rugged and have an inherent precision better than one minute of arc even in relatively unskilled hands—I do not know the American market, but an instrument of this type is available in Europe for the equivalent of about \$50. This allows nearly anyone to determine the distance of the Moon, and consequently its size; finally, through Kepler's third law, one can obtain the mass of the Earth.

The sextant is ideally suited for measuring the angle between two points. In normal use, of course, one of the two points is the local horizon, and the other the Sun or some other celestial object, and the construction of the sextant is optimized for easy manipulation when used in this way, i.e., when held vertically. In the experiment described here, the sextant is used to measure the angle between the Sun and the Moon: this requires a little practice beforehand, since the sextant will be held at some large angle to the vertical—the biggest problem is to obtain the Moon and the Sun simultaneously in the sextant's telescope.

The principle of the method is very simple. One picks a part of the month when the Moon is somewhat less than quarter, and the angle between the Sun and the Moon is measured at least twice during the course of the day. We suppose in this experiment that the Sun is so far away that its parallax over a terrestrial base line is negligible to the accuracy of our measurements, and so its direction furnishes a "fixed" direction in space with respect to which we observe the Moon using the sextant.

Consider first an ideal case in which the observer is at the equator, and let us for simplicity ignore the fact that the equatorial plane, the plane of the Earth's orbit, and that of the lunar orbit do not quite coincide. Figure 1 shows the geometrical configuration of the Moon, the Sun, and the observer at two instants of time if the Moon were stationary: it follows immediately that

$$\gamma = \beta - \alpha,$$

where α and β are the measurements of the angular separation of the Moon and the Sun at two different moments during the day. Now, the distance of the Moon is much greater than the size of the Earth (if one does not wish to know this before finding the result, one can consider what follows to be a first approximation to the distance between the Moon and the surface of the Earth, followed by an iterative calculation which will turn out to converge very rapidly); consequently, to a rather good approximation:

$$AB \cos \delta = D \sin \gamma,$$

where D is the distance of the Moon, and δ is the angle between the apparent direction of the Moon seen at B and its apparent direction when it is highest in the sky, i.e., the local meridian. Here δ appears only as a cosine; consequently, it may be obtained to a sufficient precision simply by knowing the time at which the measurement B is made, and the times at which the Moon rises and sets that day. Alternatively, it could be found using the sextant, but this requires more involved computation which will detract from the basic simplicity of the method with no significant gain in precision or interest.

AB is determined by knowing the time interval t between A and B, and the radius of the Earth R:

$$AB = 2R \sin (\pi t / 24)$$

if t is measured in hours.

Two important corrections must be applied.

(1) The Moon is not stationary, but completes an orbit around the Earth in just over 27 days. Consequently γ, which is a measure of the triangulation parallax, is not simply $\beta - \alpha$, but is given by

$$\gamma = (\beta - \alpha) - 6t/27.$$

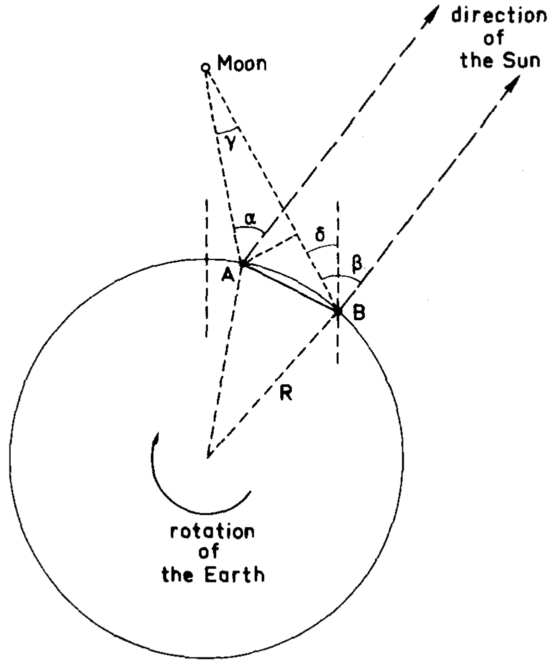

Fig. 1. Geometrical configuration of the Moon, the Sun, and the observer at two instants of time if the Moon were stationary.

The negative sign appears because the orbital motion is in the same direction as the Earth's rotation.

(2) If one is not at the equator, R must be replaced by $R \cos \lambda$, where λ is the local latitude: the observer executes around the polar axis a circle of radius smaller than that of the Earth. A subsidiary preliminary experiment with the sextant could be used to find the latitude; for midlatitude areas of the world, a high precision is not needed, since the latitude enters as a cosine.

Other corrections, due to the inclination of the Earth's axis and the lunar orbit, may be neglected; the values are small, will appear essentially as cosines, and may be expected to modify the final result by only about 10% at most.

One might suppose that an entire day is needed to obtain a reasonable result. This is not so. Consider the simplest (and most advantageous) situation in which one of the measurements is made at around the time when the Moon crosses the meridian, so that $\delta = 0$. Then for a latitude of 45°, one will obtain a parallax γ after

$$(24/\pi) \arcsin (D \sin \gamma / R\sqrt{2}) \text{ h}.$$

For a reasonable determination by unskilled students using a simple sextant whose precision is 1 arcmin, one might require γ to be at least 10 arcmin; this gives a time interval of the order of only about 1 h!

Several practical precautions should be taken to obtain reproducible results. (1) A given set of measurements should be carried out with the same sextant. All sextants have systematic errors which will be eliminated during a difference operation using the same instrument. (2) Rather than superimpose the Sun on the Moon in the sextant telescope, it is preferable to bring the limb of one in contact with the limb of the other; this is easier and more accurate and although it does not give the true angle between the Sun and the Moon, the error is eliminated when taking the difference between two measurements. (3) Most cheap sextants have some play in their gears. Consequently, one should always approach the contact point between the Moon and the Sun from the same direction and against the gear train; it is also a good idea not the change the sextant setting between two measurements since this will minimize residual play. (4) If the angular separation between the Sun and the Moon is large, one should avoid taking readings when one or the other is low in the sky: differential refraction will not be negligible.

This experiment always surprises the students who try it: after all, one does not usually associate a hand-held $50 gadget with fundamental astrometrical measurements. The experiment in fact shows the value of a "zero-measuring" device. The sextant owes its precision not merely to its 1 arcmin mechanical precision—this is actually quite easy to build in—but rather to the fact that the angle between two distinct objects is measured by superimposing their images using a single optical system: consequently, external perturbations affect both images equally and an accurate measurement can be obtained in spite of, for example, slightly shaky hands and without the need for massive supporting elements, leveling screws, etc. Moreover, by using differences between successive measurements, systematic instrumental errors are reduced or eliminated.

From the astronomical point of view, this experiment simulates the measurement of stellar parallaxes: the position of a star with respect to a background reference system of more distant "fixed" stars is measured—usually photographically—during the Earth's orbital motion around the Sun, but before obtaining a useful parallax it is necessary to allow for the intrinsic motion of the star itself. The same kinds of problems arise as in our sextant experiment; for example, one should not mix astrometrical results from different instruments.

And finally, three fundamental quantities are obtained rather easily during an afternoon's pleasant work in the sunshine!

On the approach to electro- and magneto-static equilibrium

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Many textbooks claim that the relaxation time for the approach to electrostatic equilibrium is ϵ_0/σ. We show that, for a good conductor, this claim is false. For such a conductor, the approach to the electro- and magneto-static equilibrium hinges on the damping of the induced dynamic electric and magnetic fields. The relaxation time depends on the conductivity, the geometry of the conductor, and the details of the initial charge distribution.

I. INTRODUCTION

As is well known, in a homogeneous conductor in electrostatic equilibrium, the electric field is zero and all the free charges reside on the surface. A question frequently asked in electricity textbooks is: how long does the conductor take to achieve this equilibrium configuration if the charges are initially distributed all over the volume? The authors of many introductory and even some advanced textbooks commit a glaring error in their attempts at answering this question.[1] They commonly present the following "derivation" of the characteristic relaxation time for the electrostatic field, taking as starting point the continuity equation

$$\nabla\cdot\mathbf{j} + \frac{\partial\rho}{\partial t} = 0, \tag{1}$$

Ohm's law

$$\mathbf{j} = \sigma\mathbf{E}, \tag{2}$$

and Gauss' law

$$\nabla\cdot\mathbf{E} = \rho/\epsilon_0. \tag{3}$$

They substitute Eq. (2) into Eq. (1) and then use Eq. (3) to obtain a differential equation for the charge density

$$\frac{\sigma}{\epsilon_0}\rho + \frac{\partial\rho}{\partial t} = 0, \tag{4}$$

which has the solution

$$\rho = \rho_0 e^{-(\sigma/\epsilon_0)t}. \tag{5}$$

From this, they jump to the conclusion that the relaxation time for achieving electrostatic equilibrium is

$$\tau_R = \epsilon_0/\sigma. \tag{6}$$

For a good conductor, e.g., for copper with a conductivity $\sigma = 1/(1.7\times10^{-8}\ \Omega\cdot\text{m})$, this expression yields an extremely short relaxation time, $\tau_R = 1.5\times10^{-19}$ s.

But a bit of thought immediately convinces us that this expression for the relaxation time cannot be right—it has a nonsensical dependence on the relevant physical parameters. For a very good conductor, the relaxation time ought to be very long because, in the absence of dissipative forces, the free charges will surge back and forth on the conductor instead of settling quickly into their equilibrium positions. Thus, we expect that the relaxation time ought to increase with the conductivity. We also expect, on the basis of mechanical analogies, that the relaxation time ought to increase with the size of the conductor. Equation (6) fails to

